MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2022/2023 (TERM ID 2210 & 2215)

BSP2014 – INTRODUCTION TO APPLIED PROBABILITY AND STOCHASTIC PROCESSES

(All sections / Groups)

13 FEBRUARY 2023
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **SIX (6)** printed pages inclusive of the cover page and formulae sheet.
2. Answer **ALL** four questions in answer booklet provided.
3. Students are allowed to use authorized calculators by lecturer only.
4. Marks are shown at the end of each question.

Question 1 [20 marks]

(a) The risk that a man will develop cancer of the pancreas during his lifetime is 1 in 78. Let X= the number of men you ask until one says he has pancreatic cancer.

(i) What is the probability of that you ask ten men before one says he has pancreatic cancer? [3 marks]

(ii) Find the mean and standard deviation of X. [5 marks]

(b) The times in minutes that Thomas wait for a bus follows a uniform distribution defined over the interval [3,9]. Find

(i) the expected waiting time. [2 marks]

(ii) the variance of the waiting time. [2 marks]

(iii) the probability that Thomas will take more than 7 minutes to wait for a bus. [4 marks]

(c) The amount of time a student takes to solve a statistics problem follows an exponential distribution with the average amount of time took equal to 8 minutes. What will be the probability that the student will take within 5 minutes to solve the statistics problem? [4 marks]

Question 2 [30 marks]

(a) Suppose that X is a random variable that has the following probability function:

$$f(x) = \begin{cases} 0.3 & if \quad x = 2 \\ 0.2 & if \quad x = 4 \\ 0.5 & if \quad x = 6 \end{cases}$$

(i) What is the moment generating function (MGF) for X? [3 marks]

(ii) Find the E(X) and the $E(X^2)$ using MGF. [6 marks]

(b) Given the joint probability density function:

$f(x,y) = cx^2y$ for $0 \le x \le 1$; $0 \le y \le 1$

(i) Find the constant c. [6 marks]

(ii) Find the marginal pdf of X and Y. [6 marks]

(iii) Find $P\left(Y \le \frac{1}{2}, X \le \frac{1}{2}\right)$. [4 marks]

(iv) Find $P\left(Y \le \frac{1}{2} \middle| X \le \frac{1}{2}\right)$. [5 marks]

Continued...

Question 3 [20 marks]

(a) The KLIA Express train stop by at KL Sentral station with an average of 9 trains per hour. A KLIA Express train has just left.

(i) Find the probability that the interarrival time for the next KLIA Express train takes less than 10 minutes. [5 marks]

(ii) Find the probability that passenger needs to wait more than 20 minutes for the next train to arrive. [3 marks]

(b) In a simple random walk, the chances to walk to the right are one third the chances to walk to the left. If the walker starts from origin (X_0),

(i) find p and q. [4 marks]

(ii) find $P(X_5 = -3)$. [4 marks]

(iii) find $E(X_{10})$ and Var (X_{10}). [4 marks]

Question 4 [30 marks]

Packet drinks are popular and on demand especially before Chinese New Year. Hence, Dr. Chong would like to know the preference of packet drinks among Malaysian. Suppose that X_n is a Markov Chain on 3 famous brand of packet drinks in Malaysia market namely Drinho (D), Marigold (M) and Yeo's (Y) with the following transition probability matrix:

$$P = \begin{bmatrix} 0.2 & 0.1 & 0.7 \\ 0.5 & 0.1 & 0.4 \\ 0.3 & 0 & 0.7 \end{bmatrix}$$

[Note: Leave your answer in 2 decimal places]

a) Draw the transition diagram. [4 marks]

b) Given that Dr. Chong bought Drinho in 2021, what is the probability that she will buy Yeo's in 2023? [5 marks]

c) Given initially the share for Drinho, Marigold and Yeo's is 50%, 10% and 40% respectively. Find the share of Marigold after 2 years. [5 marks]

d) Determine the stationary distribution. [13 marks]

e) Determine whether all the states are absorbing or non-absorbing states. [3 marks]

End of Questions...

FORMULAE

A. PROBABILITY DISTRIBUTION

Bernoulli Probability Distribution

$P(X = x) = p^x q^{1-x}$ for $x = 0, 1$

Binomial Probability Distribution

$P(X = x) = {}^nC_x \, p^x q^{n-x}$ for $x = 0, 1, \ldots, n$

E(X) = np ; Var (X) = np (1 – q)

Poisson Probability Distribution

$P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$ for $x = 0, 1, 2, \ldots$

E(X) = Var (X) = λ

Geometric Probability Distribution

$P(X = x) = pq^{x-1}$ for $x = 1, 2, \ldots$

E(X) = $\dfrac{1}{p}$; Var (X) = $\dfrac{1-p}{p^2}$

Uniform Probability Distribution

$f(x) = \dfrac{1}{b-a}$ for $a < x < b$

E(X) = $\dfrac{a+b}{2}$; Var (X) = $\dfrac{(b-a)^2}{12}$

Exponential Probability Distribution

$f(x) = \lambda e^{-\lambda x}$ for $x > 0$

E(X) = Var (X) = $\dfrac{1}{\lambda}$

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B. MOMENT

k^{th} Moment about the origin of X:

$$\mu'_k = E[X^k]$$

Moment Generating Function for X:

$$M_X(t) = E\left(e^{tx}\right)$$

$$E(X) = M_x^{(1)}(t=0) \quad ; \quad E(X^2) = M_x^{(2)}(t=0)$$

C. STOCHASTIC PROCESS

Simple Random Walk:

$$P(X_n = m) = \begin{pmatrix} n \\ \frac{n+m}{2} \end{pmatrix} p^{\left(\frac{n+m}{2}\right)} q^{\left(\frac{n-m}{2}\right)}$$ where m and n are both even or both odd.

$E(X_n) = n(p - q)$; $Var(X_n) = 4npq$

Gambler's Ruin Problem

$$P_a = \begin{cases} \dfrac{\left(\frac{q}{p}\right)^a - \left(\frac{q}{p}\right)^c}{1 - \left(\frac{q}{p}\right)^c}, & p \neq q \\ 1 - \dfrac{a}{c}, & p = q \end{cases} \quad ; \quad P_a + P_b = 1$$

$$D_a = \begin{cases} \dfrac{1}{q-p}\left\{a - c\left[\dfrac{1 - \left(\frac{q}{p}\right)^a}{1 - \left(\frac{q}{p}\right)^c}\right]\right\}, & p \neq q \\ a(c-a), & p = q \end{cases}$$

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D. POISSON PROCESS

Pmf for N(t) : $$P\{N(t)=n\}=\frac{e^{-\lambda t}(\lambda t)^n}{n!}$$
Pdf for T_n: $$f(t)=\lambda e^{-\lambda t}$$
Pdf for S_n : $$f(t)=\frac{\lambda e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}$$

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